Newtonian Cosmology embedded on Standard Universe: A simple solution!

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Abstract: This paper discusses the applicability of Newtonian cosmology to Relativistic cosmology. Here the 1930's work of W.H.McCrea and E.A.Milne on the relationship between the relativistic cosmology and Newtonian cosmology, and the extension of their works is in the Newtonian case each particle is steadily decelerated. In this paper we discuss the relationship among the deceleration parameter, energy, density parameter, and also curvature which are also related the standard cosmological universe.

Keywords: Newtonian Cosmology, Relativistic Cosmology, Deceleration parameter.

1 Introduction

In the 1934, E.A. Milne [1] and after publishing his paper W. H. McCrea and E.A. Milne [2] initiated an investigation into the relationship between the universe of relativistic cosmology and the universe which can be constructed using only Newtonian theory. They showed that universe is decelerated. In addition, Tipler [3] also described the decelerating Newtonian universe by using Einstein and geodesic equations. It is further shown that a space of positive, negative or zero curvature corresponding elliptic, parabolic and hyperbolic velocities respectively. In relativistic cosmology J. N. Islam [4] discussed, for zero pressure, the condition of the universe with curvature and density parameter. Here more details will be presented to enable the assumptions the condition of universe when energy is negative, zero or positive and relation among the density, deceleration parameter, and curvature and also solving another way to the relation between Newtonian and standard universe by using energy and deceleration parameter.

2 A relation between Energy and Deceleration parameter

In McCrea and Milne's paper, we are just resolved their calculation [7] and try to understand their decelerated universe. The following results obtained an earlier paper by McCrea and Milne [2] investigated on Newtonian mechanics, the velocity v is not necessarily equal to the parabolic velocity of escape and v was assumed to be the velocity of a particle at distance r from the observer at time ***t***. This velocity [5, 7] was assumed radial in nature and a function of r and ***t.*** Detailed calculation [7] of McCrea and Milne's paper [1,2] and solved that our universe is oscillating universe. The equation of motion,

$$\left.\begin{aligned} \frac{Dv}{Dt} &= -\frac{GM(r)}{r^2} \\ \frac{1}{r}\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\right) &= -\frac{4}{3}\pi G\rho \end{aligned}\right\} \quad (2.1)$$

And at last the required solution[2] is

$$t = \int_0^\theta \frac{\theta^{1/2}\, d\theta}{\left(\frac{8}{3}\pi G + A\theta\right)^{1/2}}$$

If A > 0 (hyperbolic case), as ***t*** increases, ϕ or θ steadily increases, and ρ steadily decreases to zero. If A < 0 (elliptic case) θ can never exceed $(8/3)\pi G/|A|$, and the density ρ has a lower limit, after which it increases again. We then have an oscillating universe. It should be noted that in all cases Dv/Dt is always negative, so that in the Newtonian case each particle is steadily decelerated.

Let us suppose that the universe is an expanding with constant M but where radius $R(t)$ and density $\rho(t)$ are changing in time t, as it expands.

Again

$$M = \frac{4}{3}\pi\rho(t)R^3(t) = const.$$

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It's equation of motion be

$$\ddot{R} = -\frac{4}{3}\pi G(\rho + 3p)R \quad ; \quad p = 0$$

$$\Rightarrow \ddot{R} = -\frac{4}{3}\pi G\rho(t)R(t)$$

$$\Rightarrow \frac{1}{2}\dot{R}^2 = \frac{GM}{R} + E$$

[E is integral constant which represent energy]

$$\Rightarrow \frac{1}{2}\dot{R}^2 - \frac{GM}{R} = E \qquad (2.2)$$

Where E is the total Energy.

On the other hand, for standard universe the FRW [6] model, we know the metric and we also get deceleration equation [6] but we don't use this deceleration equation.
From [6] we know the deceleration parameter

$$q = -\frac{\ddot{R}}{RH},$$ H is Hubble parameter.

Then deceleration, $q = -\frac{\ddot{R}R}{\dot{R}^2}$

$$\therefore q = \frac{1}{2} - \frac{E}{\dot{R}^2} \quad \text{[from 2.2]} \qquad (2.3)$$

$$or, \quad E = \dot{R}^2\left(\frac{1}{2} - q\right) \qquad (2.4)$$

Case:
1. $E < 0$, then $q > 1/2$
2. $E = 0$, then $q = 1/2$
3. $E > 0$, then $q < 1/2$

The first case expands to the closed universe, the second case represents a flat universe and the third case represents the open universe.

3 Discussion of assumption and results

Again the gravitational potential energy V of such a particle just arises from the matter within a sphere of radius $|r(t)|$ and …. At the origin

$$V(t) = -\frac{4}{3}\pi|r(t)|^3\rho(t)\frac{mG}{|r(t)|}$$

$$= -\frac{4}{3}\pi mG|r(t_0)|^2\rho(t)\frac{R^2(t)}{R^2(t_0)}$$

The kinetic energy of this particle

$$T = \frac{1}{2}m|\dot{r}(t)|^2 = \frac{1}{2}m|r(t_0)|^2\frac{\dot{R}^2(t)}{R^2(t_0)}$$

Therefore the total energy

$$E = T + V = \frac{1}{2}m\frac{|r(t_0)|^2}{R^2(t_0)}\left[\dot{R}^2(t) - \frac{8\pi G}{3}\rho(t)R^2(t)\right]$$

$$= \frac{1}{2}m\frac{|r(t_0)|^2}{R^2(t_0)}k \qquad (3.1)$$

Where $k = \dot{R}^2(t) - \frac{8\pi G}{3}\rho(t)R^2(t)$ (3.2)

From (2.4) and (3.1) we find that

$$\dot{R}^2\left(\frac{1}{2} - q\right) = \frac{1}{2}m\frac{|r(t_0)|^2}{R^2(t_0)}k$$

$$\Rightarrow q = \frac{1}{2}\left(1 - m\frac{|r(t_0)|^2 k}{\dot{R}^2R^2(t_0)}\right) \qquad (3.3)$$

If $k = 0$ then from equation (3.2) we get

$$\dot{R}^2(t) = \frac{8\pi G}{3}\rho(t)R^2(t)$$

$$\Rightarrow \frac{3H^2}{8\pi G} = \rho(t)$$

$$\Rightarrow \rho_c = \rho \qquad (3.4)$$

$$\therefore \Omega \equiv \frac{\rho}{\rho_c} = 1 \qquad (3.5)$$

And from equation (3.3) we find

$$q = \frac{1}{2}$$

Then we find out when

$$k = 0, \; then \quad \rho = \rho_c \;, \quad \Omega = 1 \;\; and \quad q = \frac{1}{2}$$

Which means that when k = 0 that means in flat universe deceleration parameter is ½ and present density is equal to critical density which holds relativistic universe.
If $k = -1$ then from equation (3.2) we get

$$\dot{R}^2(t) = \frac{8\pi G}{3}\rho(t)R^2(t) - 1$$

$$\Rightarrow \rho = \rho_c + \frac{3}{8\pi G}\frac{1}{R^2} \tag{3.6}$$

$$\Rightarrow \Omega \equiv \frac{\rho}{\rho_c} = 1 + \frac{3}{8\pi G}\frac{1}{R^2\rho_c} \tag{3.7}$$

Also equation (3.3), we find

$$q > \frac{1}{2}$$

that means when $k = -1$, *then* $\rho \rangle \rho_c$, $\Omega \rangle 1$ *and* $q \rangle \frac{1}{2}$

For open universe (k = -1), present density is greater than critical density and deceleration parameter is also greater than 1.

And if $k = +1$ then the equation (3.2) can be written

$$\dot{R}^2(t) = 1 + \frac{8\pi G}{3}\rho(t)R^2(t)$$

$$\Rightarrow \rho = \rho_c - \frac{3}{8\pi G}\frac{1}{R^2} \tag{3.8}$$

$$\therefore \Omega \equiv \frac{\rho}{\rho_c} = 1 - \frac{3}{8\pi G}\frac{1}{\rho_c R^2} \tag{3.9}$$

And equation (4.3), we find that

$$q \langle \frac{1}{2}$$

Also we find when

$k = +1$ *then* $\rho \langle \rho_c$, $\Omega \langle 1$ *and* $q \langle \frac{1}{2}$

Here, for a closed universe critical density is greater than present density and also deceleration parameter is less than 1.

Remark

From the dust universe we get some assumptions and which are embedded standard universe. Here we use Newtonian mechanics and also standard universe and try to overlapping this two mechanism and find out their relation. When energy increased or decreased then deceleration always holds on relativistic cosmological properties and also we do getting out the density parameter is hold present universe properties. Here we use 'm' as unit for simplicity. And this result is hold not only small region but also in large scale structure of the universe.

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